Solvay Brussels School
Economics & Management

ULB

Financial Risk Management and Governance

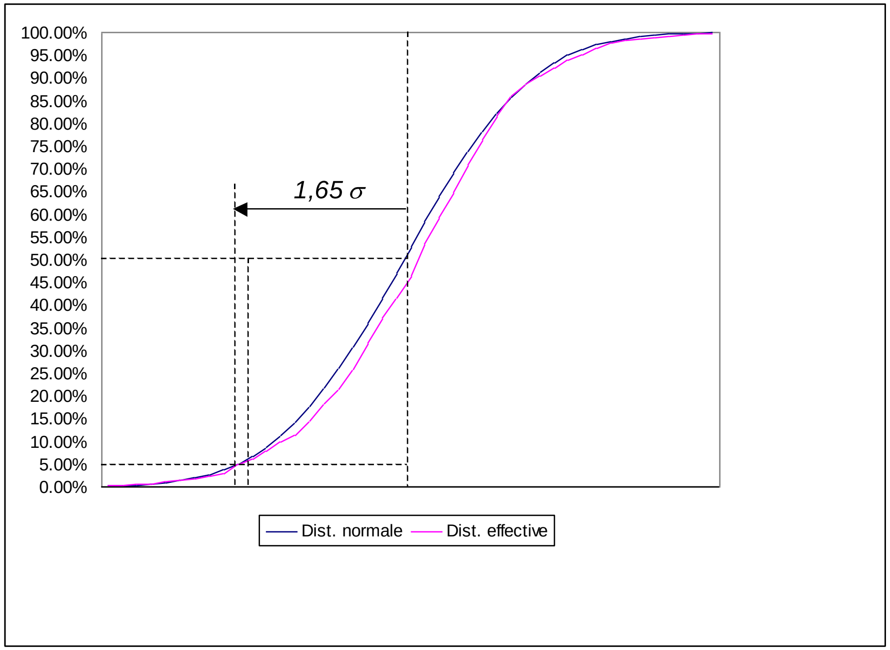
Other VaR methods

Prof. Hugues Pirotte

Idea of historical simulations…

- Why rely on statistics and hypothetical distribution?
  - Use the effective past distribution for all variables
  - Let's compare actual and normal distributions of returns

$$\Pr\left[X \le x_c\right] = \int_{-\infty}^{x_c} f(x)dx = F_X(x_c) = \alpha = 1 - c$$



Methodology

- Method
  1. Compute returns and changes for all time-series of all risk sources
     Leave them in the same order!

     m Risk sources →, n observations ↓

$$\Delta\mathbf{X} = \begin{bmatrix} \Delta X_{11} & \dots & \Delta X_{1m} \\ \dots & \Delta X_{ij} & \\ \Delta X_{n1} & \dots & \Delta X_{nm} \end{bmatrix}$$

  2. Compute the new values of the positions held in the portfolio
     a) Apply the returns to latest underlying prices to generate new price series

$$v_n \frac{v_i}{v_{i-1}}$$

     b) Reprice all positions based on those new prices
  3. Aggregate them
     » Aggregation can be an issue...
  4. Rank portfolio values in descending order
  5. Choose the desired quantile leaving c% of the values above

Ins & Outs

- you depend on a limited amount of data
- big events can
  - either not show up ⟹ P(rare events) = 0, so undervalued
  - be in the sample ⟹ can overvalue the proba of the rare event.

Accuracy

- Kendall & Stuart (1972): confidence interval for the quantile of a probability distribution estimated from sample data

$$x = q - \text{quantile of the distribution}$$

$$SE = \frac{1}{f(x)}\sqrt{\frac{q(1-q)}{n}}$$

quantile

number of obs.

⟹ quality increases by $\sqrt{n}$

where

$$n = \#obs$$

$$f(x) = \text{pdf at } x$$

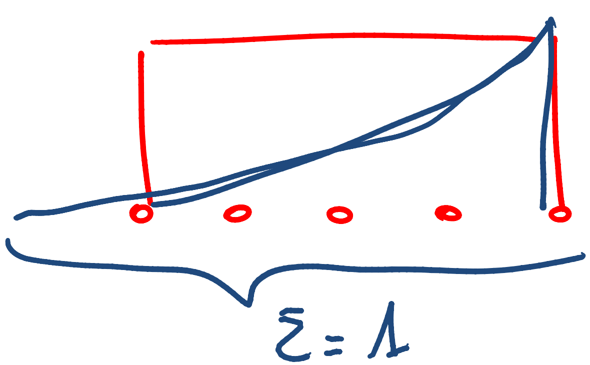
Extensions

EWMA
ARCH
GARCH

- Weighting of observations
  - Standard weights: $1/n$
  - "EWMA" idea: Boudoukh et al. (1998)
    - ✓ Weight given to change between day $n-i$ and day $n-i+1$

$$\frac{\lambda^{i-1}(1-\lambda)}{(1-\lambda^{n})}$$

    - ✓ Same as standard weighting scheme when $\lambda \to 1$
    - ✓ We sum up weights until we reach the desired quantile
    - ✓ The best value of λ can be tested using backtesting
    - ✓ The effective sample size is reduced, unless we increase substantially n.



Extensions (2)

- Incorporating volatility updating (Hull & White (1998))
  - Use of:

$$v_n \frac{v_{i-1} + \left(v_i - v_{i-1}\right)^{\sigma_{n+1}/\sigma_i}}{v_{i-1}}$$

where σ_{n+1} = current estimate of the volatility since it applies to period between today and tomorrow.

Extensions (3)

- Bootstrap
  1. We resample from the same dataset of changes to recreate many new similar datasets.
  2. The VaR is then calculated for each dataset.
  3. The confidence on the VaR is given by the range taken on the distribution of VaRs.

Monte Carlo simulations > principle

- Why rely on a single scenario? → Simulate many
  - How? → Use statistics to generate those distributional samples!
  - What is the advantage? → We can reprice everything and...

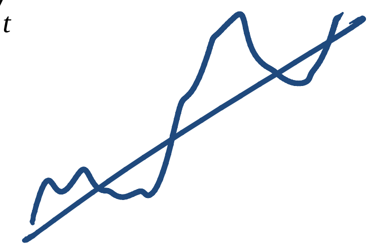
The idea of random generation

- Random returns generation…random?
  - » The Wiener process

$$\Delta W_t \sim N(0,\Delta t)$$
$$z_t \sim N(0,1)$$
$$\Delta W_t = z\sqrt{\Delta t}$$

drift diffusion

  - » The generalized Wiener process $\Delta X_t = \mu \Delta t + \sigma \Delta W_t$



  - » The Ito process $\Delta X_t = \mu(X,t)\Delta t + \sigma(X,t)\Delta W_t$

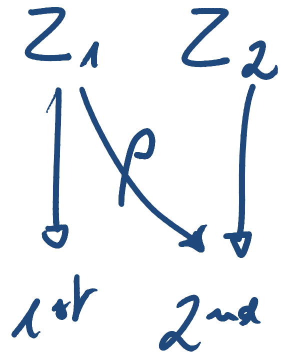
- The Geometric Brownian motion
  - » Returns distribution $\Delta S_t = \mu S \Delta t + \sigma_i z\sqrt{\Delta t}$
  - » Limit of the model $\frac{dS}{S} = d\ln(S) \Rightarrow \ln(S_T) - \ln(S_t) = \left(\mu_i - \frac{\sigma_i^2}{2}\right)\Delta t + \sigma_i z\sqrt{\Delta t}$

Ito's calculus

The Choleski decomposition

- Generating correlated randoms

$$\begin{cases} Z_1 = \eta_1 \\ Z_2 = \rho\eta_1 + \left(1-\rho^2\right)^{1/2}\eta_2 \end{cases}$$



$$\begin{bmatrix} Z_1 \\ Z_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \rho & \left(1-\rho^2\right)^{1/2} \end{bmatrix} \begin{bmatrix} \eta_1 \\ \eta_2 \end{bmatrix} \Rightarrow Z = H\eta$$

$$\rho^2 + (1-\rho^2)$$

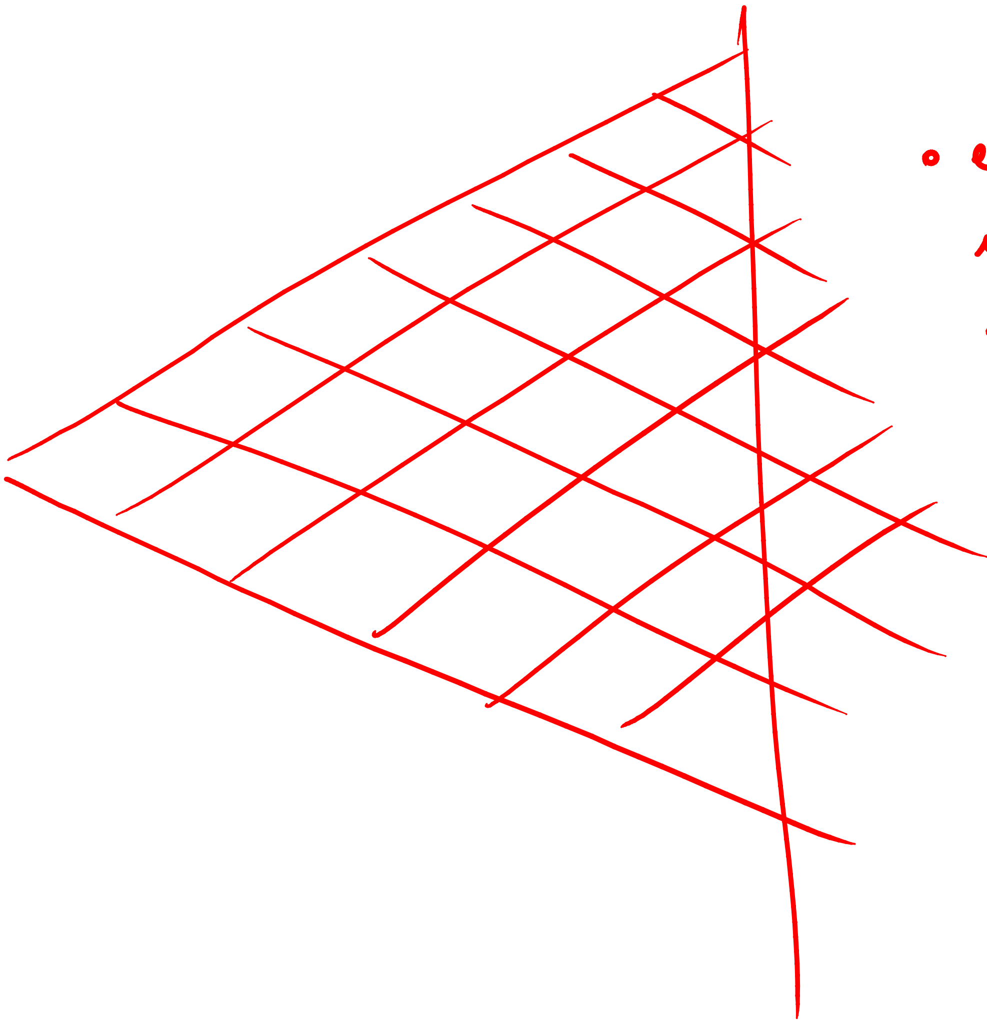
$$Var(Z) = \begin{bmatrix} \sigma_{Z_1}^2 & \sigma_{Z_1,Z_2} \\ \sigma_{Z_1,Z_2} & \sigma_{Z_2}^2 \end{bmatrix} = \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix} = V$$

$$V = HH' = \begin{bmatrix} 1 & 0 \\ \rho & \left(1-\rho^2\right)^{1/2} \end{bmatrix} \begin{bmatrix} 1 & \rho \\ 0 & \left(1-\rho^2\right)^{1/2} \end{bmatrix}$$

$$Var(\eta) = 1 \quad \text{et} \quad Z = H\eta$$

$$Var(Z) = \mathrm{E}\left[(H\eta)(H\eta)'\right] = \mathrm{E}\left[H\eta\eta' H'\right] = H\,\mathrm{E}\left[\eta\eta'\right]H' = H\,I\,H' = HH'$$

Binomial methods (tree methods)



(+)

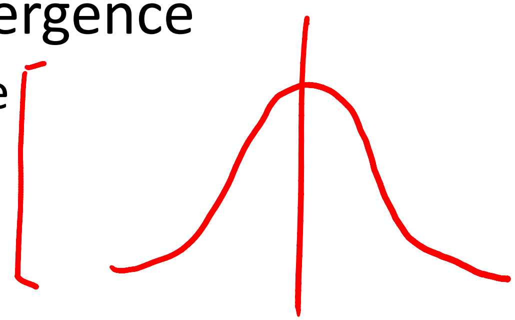
- easier to directly identify all potential cases including extremes

(−)

- hard to represent when more than 1 or 2 variables.

Accuracy of simulations

- The effect of sampling variability
  - » the empirical distribution of ST is only an approximation, unless the number of simulations (*k*) is extremely large
  - » Monte Carlo implied independent draws
    - ✓ The standard error of statistics is inversely related to $\sqrt{k}$
- Methods to speed up convergence
  - » Antithetic variable technique



reduce "noise" due to simulation
increase speed
have symmetry

  - » Control variate technique

1'000 simulations
500 really generated
copy them & change the sign

  - » Quasi-random sequences (= QMC)

Comparison of models

	Delta-Normal (or var-covar)	Historical Simulation	MonteCarlo Simulation
Valuation	Linear (Local)	Full	Full
Distribution ▪ Shape ▪ Extreme events	 → Normal → Low probability	 → Actual → In recent data	 → General → Possible
Implementation ▪ Ease of computation ▪ Communicability ▪ VaR precision ▪ Major pitfalls	 → Yes → Easy → Excellent → Non-linearities, fat tails	 → Intermediate → Easy → Poor with short window → Time variation in risk, unusual events	 → No → Difficult → Good with many iterations → Model risk

Inspired from Jorion, *Financial Risk Manager Handbook*

References

- Prof. H. Pirotte
- Some excerpts from:
  - Hull (2007), “Risk management and Financial Institutions”
  - The RiskMetrics technical document
  - Jorion (2008), “Financial Risk Manager Handbook”
- Others:
  - Jorion (2000), « Risk Management Lessons from LTCM ».